Comments:

[1] ~~This getting $G = -D^T$ (or vice versa) is tedious but it gives some advantages~~

~~Gresho's Book ← mathematical~~

[2] Saves space and work in e3xxx.f xGoC [D:C] → stored in LHSP

$\Delta \dot{U}$ $\Delta \dot{P}$

all element level computations take place in these files

2/20/09

[3] Later we will see that we use iterative solvers
⇒ matrix vector products

$\vec{q} = \underset{\approx}{A}\vec{P}$

$$\begin{bmatrix} K & G \\ D & C \end{bmatrix} \begin{bmatrix} \Delta \dot{U} \\ \Delta \dot{P} \end{bmatrix}$$

this shape

How do we do that if we did not store G?
→ Hardwire the Ap product to use $-D^T$

$$\underset{\approx}{K}\,\Delta\tilde{\dot{U}} + \underset{\approx}{G}\,\Delta\tilde{\dot{P}} = \underset{\approx}{K}\,\Delta\tilde{\dot{U}} - \underset{\approx}{D}^T\Delta\tilde{\dot{P}}$$

[4] Note the N_i^a, the i is just there to help us place the term in the 3X3 K_{ij}^{ab} matrix or D_j (1X3 matrix)

~~We use the same shape function regardless of the value of i. Furthermore, N_p^a is also the same for stabilize methods like we discussed before.~~

There are FEM methods, mixed methods, where the polynomial order of velocity is higher than pressure.

\boxed{}

[5] It is fairly common when studying incompressible flow to study an auxilliary equation that is derived from the already discretized system,

$$\begin{bmatrix} \underset{\approx}{K} & \underset{\approx}{G} \\ -\underset{\approx}{G}^T & \underset{\approx}{C} \end{bmatrix} \begin{bmatrix} \Delta \dot{\underset{\sim}{U}} \\ \Delta \dot{\tilde{P}} \end{bmatrix} = \begin{bmatrix} -\tilde{\underset{\sim}{R}}^m \\ -\tilde{\underset{\sim}{R}}^c \end{bmatrix}$$ ① Fully coupled momentum-continuity system FCMC

Extract momentum equation out

$$\underset{\approx}{K} \Delta \dot{\underset{\sim}{U}} + \underset{\approx}{G} \Delta \dot{\tilde{P}} = -\tilde{\underset{\sim}{R}}^m$$

imagne we know $\underset{\approx}{K}^{-1}$

$$\Delta \dot{\tilde{\underset{\sim}{U}}} = \underset{\approx}{K}^{-1}\left[-\tilde{R}^m - \underset{\approx}{G} \Delta \dot{\tilde{P}}\right]$$ → Plug $\Delta \dot{\tilde{U}}$ into continuity eq.

Second cont

$$-\underset{\approx}{G}^T \Delta \dot{\tilde{\underset{\sim}{U}}} + C \Delta \dot{\tilde{P}} = -\tilde{R}^c$$

$$-\underset{\approx}{G}^T \underset{\approx}{K}^{-1}\left[-\tilde{\underset{\sim}{R}}^m - \underset{\approx}{G} \Delta \dot{\tilde{P}}\right] + C \Delta \dot{\tilde{P}} = -\tilde{R}^c$$ ← rearrange

$$\underbrace{\left[\underset{\approx}{G}^T \underset{\approx}{K}^{-1} \underset{\sim}{G} + \underset{\approx}{C}\right]}_{} \Delta \dot{P} = -\tilde{R}^c - \underset{\approx}{G}^T \underset{\approx}{K}^{-1} \tilde{R}^m$$

Discrete pressure poission equation $\left(\nabla^2 P\right) = \nabla \cdot \text{mom}$
we don't work w/ this

DPPE

Instead, we work with the DPPE

The incompressible code (PHASTA) calls a third party library that solves DPPE as a preconditioner to FCMC.

Why not just solve FCMC?

Eigenvalue spread between continuity equation ($\Delta \dot{P}$) and momentum equation ($\Delta \dot{U}$) so large that FCMC would have to be solved to very tight tolerances.

The "solution" solve DPPE (worse tolerance) then solve FCMC to loose tolerance but "constrained" to not violate DPPE solution

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 2 & 3 \\ 2 & 4 & 5 & 6 \\ 3 & 7 & 8 & 9 \end{array}$$

in code

[6] $K_{ij}^{eab} = \text{xkebe}(\overbrace{1:nel}^{e}, \overbrace{1:9}^{ij}, \overbrace{1:nshgl}^{a}, \overbrace{1:nshgl}^{b})$

nshgl = number of shape functions in the element

i.e. tet, linear = 4

Assemble this   fillsparse

xkebe Assemble $\Rightarrow$ lhsK(1:nnz-tot, 9)

the number of non-zeros in the matrix

-i

xGoC $\Rightarrow$ lhsP(1:nnz_tot, 4)

[7] Above we mentioned $\underset{\sim}{K}^{-1}$. We can't really get this but instead we use $\underset{\sim}{K}^{-1} \approx \hat{\underset{\sim}{K}}^{-1}$ typically $\underset{\sim}{K} = \underset{\sim}{K}^{l}$ lumped diagonal of the real k.

~~Trivial to invert~~

Note: $\exists$ something sort of in between compressible and incompressible flow.

$\Rightarrow$ pseudo-compressible flow

only really sensible for acoustics

The idea is to put $\frac{\partial \rho}{\partial t}$ back into continuity equation.

$$\frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial P}\frac{\partial P}{\partial t}$$

$$\alpha \quad \boxed{\frac{\partial \rho}{\partial P}}\frac{\partial P}{\partial t} + \rho u_{i,i} = 0$$

$$\boxed{\frac{\alpha}{\rho}\frac{\partial P}{\partial t}} + u_{i,i} = 0$$

Extra time term in continuity equation.

~~Probably~~ push $\alpha_{f_p} = \alpha_{f_u}$

$\alpha_{m_p} = \alpha_{m_u}$

$\gamma_p = \gamma_u$

This technique is also pursued as a relaxation technique.

$$\frac{\partial P}{\partial t} \rightarrow \frac{\partial P}{\partial \tau}$$

$\leftarrow$ pseudo time